

Our Friend the All-Pass

Preface

Most effects developers first run into the all-pass structure when learning about reverb algorithms. While the all-pass is very useful in this case for its impulse response understanding its phase response will allow users to better use the structure in other ways.

The First Order All-Pass

The following figure shows the typical structure of a first order all-pass as we typically implement it in the FXCore using the APMA and APMB instructions:



While many programmers have implemented this structure in their reverb and phaser programs they typically set the coefficient K based on how things sound with no understanding of what it means.

An all-pass filter passes all frequencies without attenuating any of them, rather it shifts the phase of the frequencies by 0 to -180 degrees. The shift is 0 degrees at 0Hz and -180 degrees at (Fs/2)Hz, K sets the frequency, Fc, where the -90 degree shift will happen. K is calculated from:

$$K = \frac{\tan\left(\pi * \frac{Fc}{Fs}\right) - 1}{\tan\left(\pi * \frac{Fc}{Fs}\right) + 1}$$

If we were to set Fc to 300Hz we would see a result such as:





where we can see that the magnitude response is flat across all frequencies but the phase response is shifted from 0 to -180 degrees and the -90 phase shift occurs at 300Hz.

Understanding the simple facts that a first order all-pass does not affect the magnitude response, the phase will always change from 0 to -180 degrees and knowing how to calculate K and thus controlling the frequency of the -90 degree shift a DSP programmer can create many different effects both common and uncommon.

It is VERY important to note that the all-pass can create a large internal gain that can exceed +/-1.0 and therefore it is not uncommon to have to lower the signal level prior to any all-pass stages and raise it up after.

The All-Pass Low-Pass Filter

Low-pass and high-pass filters are two of the most common structures in audio DSP. Most people use a simple IIR type filter as its step response is the same as the simple analog low pass filter. The all-pass can be used to create a low-pass if we consider the following:

If we add a signal to its self the result is a signal twice as large as the original.

If we add a signal to an inverted version (phase shift of 180 degrees) of its self the result is 0.

As we can see in the graph above, low frequencies are basically in phase and higher frequencies are out of phase so if we put a signal through the all-pass and add it back to its self we would have a low-pass type filter response.

Therefor the all-pass low-pass would look like:





And its frequency and phase response would look like:



Note that the output is +6db since the signal is double the input so we would typically scale the input signal by 0.5 to account for this.

Also note that the frequency of the -90 degree phase shift as set by K is the -3db frequency in this filter so K sets the -3db point of the all-pass low-pass just like you would set the -3db frequency in any other filter.

Why does the K coefficient set the -3db point of the all-pass low-pass? As it turns out if you add a signal to a -90 degree shifted version of itself you get a result that is 3db lower than adding the signal to an in phase version of itself.

The All-Pass High-Pass

Based on the above low-pass, we can deduce that if we subtract the output of the first order allpass from the input signal we will get a high pass result. So simply changing the final addition to a subtraction like:





Would result in a frequency and phase response like:



Note again the +6db gain and that the -3db point is set by K

Serial First-Order All-Pass

The all-pass structure is used in phase shifting pedals since that is exactly what it does, it shifts the phase of the signals but in phase shifting pedals we always use pairs of all-pass filters. If we think about what the all-pass does we can understand why we use pairs. The all-pass shifts the phase of a signal by 0 to -180 degrees, if it is followed by a second all-pass then this all-pass adds an additional shift of 0 to -180 degrees to the signal or a total shift of 0 to -360 degrees.





As -360 degrees is the same as 0 degrees we now have a structure that shifts from 0 to -180 and back to 0 degrees. If we then add this result to the original signal we will see a notch at the - 180 degree point which is set by K, note all stages use the same value for K.



Now something interesting happens if we add an additional 2 stages of all-pass for a total of 4 stages of all-pass, after summing the original signal back in we will see 2 notches but neither of them at the frequency set by K. To understand this we need to consider what is happening in each all–pass and the fact that to produce the notch we need to sum the original signal with a copy of its self where a frequency has a 180 phase shift. For a 2 stage all-pass this is simply the 90 degree phase shifted signal as it is shifted by 90 degrees twice and results in a 180 degree phase shift and hence will cause the notch when summed with the original signal.

With the 4 stage all-pass we will be causing the notch where the signal has a 45 degree phase shift in each stage as 4*45=180.





That explains the first notch and why it is not at Fc but what about the second notch? This second notch will happen where the phase shift is 135 degrees in each stage because 135*4 = 540 degrees, 540 - 360 = 180 degrees.

Every time you add an additional 2 stages of all-pass you will add an additional notch and you will shift the frequencies of these notches.

An important fact here is that the locations of the notches are not harmonically related and this is what gives a phase shifter its unique sound.

Simplifying K Calculation

Examining the equation for K above, one might feel solving an equation like this in the FXCore will be rather difficult as FXCore does not have a TAN function built in. But if we consider that the range of FC/FS is 0 to 0.5 we see that K ranges from -1 to +1 in an almost linear fashion, this range can be easily calculated in FXCore from a POT or other source such as a SINE wave.

The Second-Order All-Pass

The second order all-pass basically wraps an all-pass around another all-pass. In the case of the second order all-pass the phase shifts from 0 to -360 degrees rather than 0 to -180 degrees.





Unlike cascaded first order all-pass the K coefficients in the second order all-pass are independent and each effects a different aspect of the filter.

Ki the coefficient of the inner all-pass filter sets the -180 degree frequency of the structure while the outer coefficient Ko sets the transition range (bandwidth between -90 and -270 degrees) of the structure.



The calculation of Ko and Ki differ from the calculation of K for the first order case:

$$Ko = \frac{1 - tan\left(\pi * \frac{BW}{Fs}\right)}{1 + tan\left(\pi * \frac{BW}{Fs}\right)}$$



$$Ki = -cos(2 * \pi * \frac{Fc}{Fs})$$

Since Ko and Ki are independent we can set the center frequency and bandwidth independently. However this means that Q, which is defined as Fc/BW, is not constant as Fc is varied.

Second-Order All-Pass Notch and Bandpass Filters

The second order all-pass can be easily made into a notch or bandpass filter by simply adding or subtracting the input signal from the output of the 2nd order all-pass.



Like the first-order all-pass the internal gains of the second-order all-pass can be quite large and scaling the input and output to eliminate internal clipping will be required.

The All-Pass as a Non-Linear Delay

If we put a signal through a single delay then all frequencies are delayed the same amount of time and the phase shift is linear from 0 to -180 degrees:







Since we can adjust the -90 degree location of a first-order all-pass we are actually changing the delay time for different frequencies therefor the delay time is no longer constant for all frequencies and the phase shift is not linear, it depends on K:



So what does this mean? It means that for a single delay the signal going in is the same as the signal coming out just delayed in time. For an all-pass it means that the signal going in is not the same as the signal coming out, it will contain all the same frequencies and magnitudes but due to the delay through the all-pass being different for different frequencies the output may look very different.

As an example, if we put a 500Hz squarewave through a first-order all-pass with Fc=1KHz we get a signal that looks like:





But the FFT of the original square wave and the result of the all-pass are the same, just the phase relationship between the frequencies has changed.







As Fc is increased the result will look more like a squarewave as more of the harmonics experience less of a delay and as a result maintain their phase relationship.

Experimental Noize Inc. reserves the right to make changes to, or to discontinue availability of, any product or service without notice.

Experimental

Noize

Experimental Noize Inc. assumes no liability for applications assistance or customer product design. Customers are responsible for their products and applications using any Experimental Noize Inc. product or service. To minimize the risks associated with customer products or applications, customers should provide adequate design and operating safeguards.

Experimental Noize Inc. make no warranty, expressed or implied, of the fitness of any product or service for any particular application.

In no even shall Experimental Noize Inc. be liable for any direct, indirect, consequential, punitive, special or incidental damages including, without limitation, damages for loss and profits, business interruption, or loss of information arising out of the use or inability to use any product or document, even if Experimental Noize Inc. has been advised of the possibility of such damage.

SAFETY-CRITICAL, MILITARY, AND AUTOMOTIVE APPLICATIONS DISCLAIMER: Experimental Noize Inc. products are not designed for and will not be used in connection with any applications where the failure of such products would reasonably be expected to result in significant personal injury or death ("Safety-Critical Applications"). Safety-Critical Applications include, without limitation, life support devices and systems, equipment or systems for the operation of nuclear facilities and weapons systems. Experimental Noize Inc. products are not designed nor intended for use in military or aerospace applications or environments. Experimental Noize Inc. products are not designed nor intended for use in automotive applications.

Experimental Noize Inc. Scottsdale, AZ USA www.xnoize.com sales@xnoize.com